

JEE QUESTIONS - WORK, ENERGY AND POWER

- 1) A particle moves along the curve  $y=x^2$  in the  $xy$  plane from point  $A(0,0)$  to  $B(1,1)$ . A force  $\vec{F} = (2x\hat{i} + 3y^2\hat{j})$  act on the particle. Find the work done by  $\vec{F}$  along the path

ANSWER

Parametrize path by  $x=t, y=t^2$  for  $t \in [0,1]$

$$\begin{aligned}\text{Displacement element } (d\vec{r}) &= (dx)\hat{i} + (dy)\hat{j} \\ &= (dt)\hat{i} + (2t^2)\hat{j} \\ &= (1t)\hat{i} + (2tdt)\hat{j}\end{aligned}$$

$$W = \vec{F} \cdot \vec{s}$$

$$\begin{aligned}&= [2t] \cdot [dt] + [3(t^2)] \cdot [2tdt] \\ &= 2tdt + 3t^4 \cdot 2tdt\end{aligned}$$

$$W = 2tdt + 6t^5dt$$

Integrate  $t$  from 0 to 1

$$W = \int_0^1 (2t + 6t^5) dt = [t^2 + t^6]_0^1$$

$$= 1 + 1$$

$$= \underline{2J}$$

2. A particle of mass  $m$  moves along  $x$  under a force  $F(x) = -k/x^2$  ( $x > 0, k > 0$ ). It starts from rest at  $x=a$  and moves to  $x=b$  where  $b < a$ . Find its speed at  $x=b$

ANSWER

Force is conservative.

$$\begin{aligned}\text{Potential energy } U(x) &= -\int F(x) dx \\ &= -\int (-k/x^2) dx \\ &= -k\left(\frac{1}{x}\right) + C\end{aligned}$$

Take  $U(\infty) = 0$  yield constant accordingly but we require only differences

change in P.E from a to b

$$\Delta U = U(b) - U(a) = -K \left( \frac{1}{b} - \frac{1}{a} \right)$$

Since initial K.E is 0, mechanical energy conservation

$$\frac{1}{2}mv^2 + U(b) = U(a)$$

$$\frac{1}{2}mv^2 = U(a) - U(b)$$

$$v^2 = \frac{2K}{m} \left( \frac{1}{b} - \frac{1}{a} \right)$$

$$v = \sqrt{\frac{2K}{m} \left( \frac{1}{b} - \frac{1}{a} \right)}$$

3. A hoist lifts a mass of 500kg at constant speed 2m/s. The motor draws electrical power  $P_{in} = 15\text{KW}$ . Determine the mechanical power output, the useful power & efficiency.

ANSWER

Useful mechanical power is

$$\begin{aligned} P_{out} &= Mg v \\ &= 500 \times 10 \times 2 \\ &= 10000\text{W} \\ &= \underline{10\text{KW}} \end{aligned}$$

Input  $P_{in} = 15\text{KW}$

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} = \frac{10}{15} = \frac{2}{3} = 0.667$$

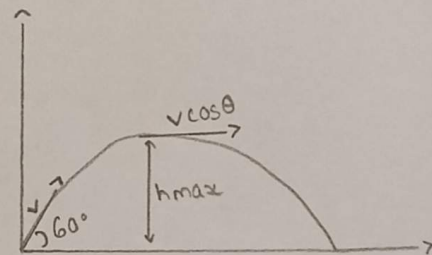
$$\Rightarrow \underline{\eta = 66.7\%}$$

Q A body of mass 2kg begins to move under the action of a time dependent force given by  $\vec{F} = (6t\hat{i} + 6t^2\hat{j}) \text{ N}$ . The power developed by the force at the time 't' is given by?

$$\begin{aligned}\text{Sol: } \vec{F} &= 6t\hat{i} + 6t^2\hat{j} \\ \vec{F} &= m\vec{a} = 6t\hat{i} + 6t^2\hat{j} \\ \vec{a} &= \frac{\vec{F}}{m} = 3t\hat{i} + 3t^2\hat{j} \\ \vec{v} &= \int_0^t \vec{a} dt = \frac{3t^2}{2}\hat{i} + t^3\hat{j} \\ P &= \vec{F} \cdot \vec{v} = (9t^3 + 6t^5) \text{ W}\end{aligned}$$

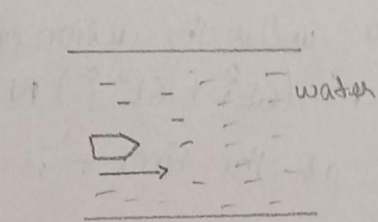
Q A ball is projected with Kinetic energy  $E$ , at an angle  $60^\circ$  to the horizontal. The kinetic energy of this ball at the highest points of its flight will become?

$$\begin{aligned}\text{Sol: } K \cdot E &= E = \frac{1}{2}mv^2 \\ \text{At highest point} \\ K \cdot E' &= \frac{1}{2}mv^2 \cos^2\theta \\ &= \frac{1}{2}mv^2 \times \frac{1}{4} \\ &= \frac{E}{4}\end{aligned}$$



Q A bullet of mass 200g having initial kinetic energy 90J is shot inside a long swimming pool as shown in the figure. If its kinetic energy reduces to 40J within 1s, the minimum length of the pool, the bullet has to travel so that it completely comes to rest is.

Sol:



$$\frac{1}{2} m x^2 = 90$$

$$\Rightarrow \frac{1}{2} \times 0.2 \times x^2 = 90$$

$$x^2 = 900$$

$$x = 30 \text{ m/s}$$

$$\frac{1}{2} m v^2 = 40$$

$$\Rightarrow \frac{2}{3} \times 30 = v$$

$$v = 20 \text{ m/s}$$

$$20 = 30 - a \times 1 \Rightarrow a = -10 \text{ m/s}^2$$

$$0 - x^2 = 2as$$

$$s = \frac{x^2}{2a} = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$$

# JEE Questions

## Work Energy And Power

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(1)

Q.1) The ratio of powers of two motors is  $\frac{3Jx}{Jx+1}$ , that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of  $x$  will be :- [JEE-Main-2023]

(1) 2 (2) 4 (3) 2.4 (4) 16

Ans:-1) To solve this question, first calculate the work done in both cases:-

Case:- I

$$m = 300 \text{ kg}; t = 5 \text{ min}; g = 10 \text{ m/s}^2$$

$$h = 100 \text{ m}$$

$$W_1 = mgh$$

$$\Rightarrow (300)(10)(100)$$

$$\Rightarrow 300000 \text{ J}$$

$$P_1 = \frac{dW_1}{dt} \Rightarrow \frac{300000}{5}$$

$$\Rightarrow 60,000 \text{ W}$$

Case:- II

$$m = 50 \text{ kg}; t = 2 \text{ min}; g = 10 \text{ m/s}^2$$

$$h = 100 \text{ m}$$

$$W_2 = mgh$$

$$\Rightarrow (50)(10)(100)$$

$$\Rightarrow 50000 \text{ J}$$

$$P_2 = \frac{dW_2}{dt} \Rightarrow \frac{50000}{2} \Rightarrow 25,000 \text{ W}$$

Given that:- the ratio of these powers is:-

$$\frac{P_1}{P_2} = \frac{3Jx}{Jx+1}$$

$$\frac{60,000}{25,000} = \frac{3Jx}{Jx+1}$$

$$12Jx+12 = 15Jx$$

$$12 = 3Jx$$

$$Jx = 4$$

Squaring both sides;

$$\therefore x = 16$$

Hence, option (4) 16 is correct.

Q.2) A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 m/s. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now? [JEE Main-2021] ②

Ans: - ② Given: -  $m = 0.5 \text{ kg}$ ,  $v = 20 \text{ m/s}$ ; using this we can calculate initial kinetic energy: -

$$K_i = \frac{1}{2}mv^2 \Rightarrow \frac{1}{2} \times 0.5 \times 400 \Rightarrow \frac{400}{4} \Rightarrow \underline{\underline{100 \text{ J}}}$$

$\Rightarrow$  Now it's told that after deflection it moves with 5% of  $K_i$ . So: -

$$K_f \Rightarrow 5\% \text{ of } 100 \Rightarrow \frac{5}{100} \times 100 \Rightarrow 5 \text{ J}$$

$$\text{Therefore: - } K_f = \frac{1}{2}mv^2$$

$$5 = \frac{1}{2} \times \frac{1}{2} v^2$$

$$\therefore v = \sqrt{20}$$

$$\therefore v \Rightarrow \underline{\underline{4.47 \text{ m/s}}}$$

Q.3) A body of mass 1 kg begins to move under the action of a time dependent force,  $\vec{F} = (t\hat{i} + 3t^2\hat{j}) \text{ N}$ . Where  $\hat{i}$  and  $\hat{j}$  are the unit vectors along x and y axis. The power developed by above force, at the time  $t = 2 \text{ sec}$ , will be — W. [JEE-Main: - 2023]

Ans: - ③ We are given that mass is 1 kg and force is;  $\vec{F} = (t\hat{i} + 3t^2\hat{j})\text{N}$ ; Power at  $t = 2\text{sec}$ .

③

$$\begin{aligned}\vec{F} &= m\vec{a} \\ \vec{F} &= (1)\vec{a} \\ \vec{F} &= \vec{a}\end{aligned}$$

$$m\vec{a} = (t\hat{i} + 3t^2\hat{j})$$

$$\frac{d\vec{v}}{dt} = (t\hat{i} + 3t^2\hat{j})$$

$$d\vec{v} = dt(t\hat{i} + 3t^2\hat{j})$$

$$\int_0^{\vec{v}} d\vec{v} = \int_0^t t\hat{i} + \int_0^t 3t^2\hat{j}$$

$$\therefore \vec{v} = \frac{t^2}{2}\hat{i} + \frac{3t^3}{3}\hat{j}$$

$\vec{v}$  = we can substitute into Power Equation:-

$$\begin{aligned}P &= \vec{F} \cdot \vec{v} \\ &\Rightarrow (t\hat{i} + 3t^2\hat{j}) \cdot \left(\frac{t^2}{2}\hat{i} + t^3\hat{j}\right)\end{aligned}$$

$$\Rightarrow \frac{t^3}{2} + 3t^5 \quad \text{--- (1)}$$

Now put  $t = 2\text{sec}$  into (1);

$$\Rightarrow \frac{(2)^3}{2} + 3(2)^5$$

$$\Rightarrow \frac{8}{2} + 3 \times 32$$

$$\Rightarrow 4 + 96$$

$$\boxed{P \Rightarrow 100\text{W}}$$

## Work, Energy and Power

Q.1. A block of mass 2kg is moving with a velocity of 5m/s on a horizontal surface. It is brought to rest by a constant frictional force. If the coefficient of friction is 0.4, find the distance traveled by the block before it comes to rest

Ans.1. • The initial kinetic energy  $K_i = \frac{1}{2}mv^2$ , where

$m$  is mass and  $v$  is velocity

$$K_i = \frac{1}{2} \times 2 \times (5)^2 = \underline{25J}$$

• The frictional force  $f = \mu N$ , where  $\mu$  is the coefficient of friction and  $N$  is normal force.

$$f = 0.4 \times 2 \times 10 = \underline{8N}$$

• The work done by friction  $W = f \times s$ , where  $s$  is the distance traveled.

$$W = K_i - K_f = 25 - 0 = \underline{25J} \text{ (since the block comes to rest)}$$

• The distance traveled  $s = W/f$

$$s = \frac{25}{8} = \underline{\underline{3.125m}}$$

$\therefore$  The final answer is 3.125m.

Q.2. A force of  $10\text{ N}$  is applied to a block of mass  $5\text{ kg}$ , displacing it by  $2\text{ m}$  in the direction of the force. If the block is initially at rest, find its final velocity.

Ans.2. The work done  $W = F \times s$ , where  $F$  is the force and  $s$  is the displacement.

$$W = 10 \times 2 = \underline{20\text{ J}}$$

• The final kinetic energy  $K_f = W + K_i$ , where  $K_i$  is the initial kinetic energy ( $0$ , since the block is initially at rest).

$$K_f = 20 + 0 = \underline{20\text{ J}}$$

• The final velocity  $v = \sqrt{\frac{2K_f}{m}}$ .

$$v = \sqrt{\frac{2 \times 20}{5}} = \sqrt{8} = \underline{2\sqrt{2}\text{ m/s}}$$

$\therefore$  The final answer is  $2\sqrt{2}\text{ m/s}$

Q.3. A pump to lift 100 kg of water to a height of 10 m in 20 s. Find the power of the pump and the work done by it.

Ans. 3. The work done  $W = mgh$ , where  $m$  is mass,  $g$  is acceleration due to gravity, and  $h$  is height

$$W = 100 \times 10 \times 10 = \underline{\underline{10000 \text{ J}}}$$

• The power  $P = \frac{W}{t}$ , where  $t$  is time.

$$P = \frac{10000}{20} = \underline{\underline{500 \text{ W}}}$$

$$\therefore \text{Work done} = \underline{\underline{10000 \text{ J}}}$$

$$\text{Power} = \underline{\underline{500 \text{ W}}}$$

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Q.1. A body of mass ~~0.5~~ 0.5 kg travels on straight line path with velocity  $v = (3x^2 + 4) \text{ m/s}$ . The network done by the force during its displacement from  $x = 0$  to  $x = 2 \text{ m}$  is

Ans. At  $x = 0$ ,  $v_1 = 4 \text{ m/s}$   
At  $x = 2$ ,  $v_2 = 16 \text{ m/s}$

$$\begin{aligned} W &= KE_f - KE_i \\ &= \frac{1}{2} (m) (v_2^2 - v_1^2) \\ &= \frac{1}{2} (2 \times 0) = \underline{\underline{60 \text{ J}}} \end{aligned}$$

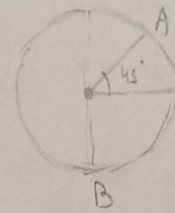
Q.2. If a rubber ball falls from a height  $h$  & rebounds upto the height of  $h/2$ . The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground are:-

Ans. Velocity just before collision:  $\sqrt{2gh}$   
Velocity just ~~before~~ after collision:  $\sqrt{2g \frac{h}{2}}$

$$\begin{aligned} \therefore \Delta KE &= \frac{1}{2} m(2gh) - \frac{1}{2} mgh \\ &= \frac{1}{2} mgh \end{aligned}$$

$$\% \text{ loss in energy} = - \frac{\Delta KE}{\Delta KE_i} \times 100 = \underline{\underline{50\%}}$$

Q.3 A body of  $m$  kg slides from the rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is:-  
 [Given  $R=14\text{m}$ ,  $g=10\text{m/s}^2$ ,  $\sqrt{2}=1.4$ ]



Ans. Applying work energy theorem from A to B:-

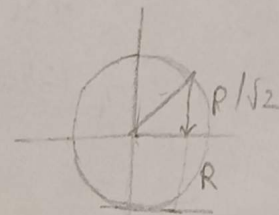
$$W_{mg} = K_B - K_A$$

$$\Rightarrow mg \left( \frac{R}{\sqrt{2}} + R \right) = \frac{1}{2} m v_B^2 - 0 \quad [\text{at rest}]$$

$$\therefore v_B = \sqrt{gR^2 \left( \frac{\sqrt{2}+1}{\sqrt{2}} \right)}$$

$$= \sqrt{\frac{10 \times 196 \times 1.8}{1.4}}$$

$$\therefore \underline{v_B \approx 22\text{m/s}}$$



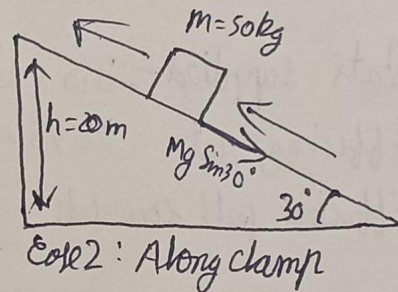
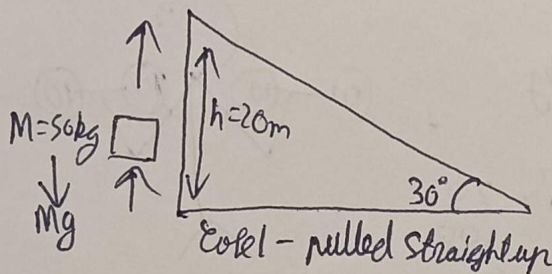
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# Work, Energy And Power

Q) A body of mass 50kg lifted to height of 20m from ground in two different ways as shown in figure. The ratios of work done against gravity in both respective cases will be:

{JEE Mains 2024(online) 5th April Morning shift}

- (A) 2:1    (B)  $\sqrt{3}:2$     (C) 1:1    (D) 1:2



Sol.

We know

$$W = \vec{F} \cdot \vec{s}$$

In this two cases,

object travels 20m vertically but through diff ways

so for both cases  $h = 20\text{m}$

$$PE_1 = mgh = 20mg, PE_2 = mgh = 20mg$$

$PE_1 : PE_2$  {Work done against gravity is gravitational potential energy}

$$20mg : 20mg$$

$$\therefore \text{ratio} \rightarrow 1:1$$

Ans: (C) 1:1

Q. A person trying to loose weight by burning fat lifts a mass of 10 kg upto height of 1m 1000 times. Assume that potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering work done only when weight is lifted up? Fat supplies  $3.8 \times 10^7 \text{ J}$  of energy per kg which is converted to mechanical energy with 20% efficiency rate. Take  $g = 9.8 \text{ m/s}^2$ . [JEE mains 2016 (offline)]

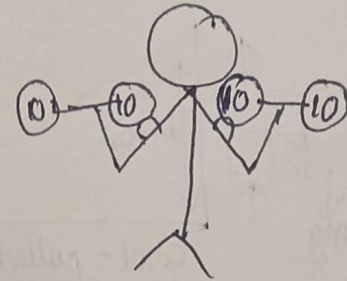
- (A)  $9.89 \times 10^{-3} \text{ kg}$  (B)  $12.89 \times 10^{-3} \text{ kg}$  (C)  $2.45 \times 10^{-3} \text{ kg}$   
(D)  $6.45 \times 10^{-3} \text{ kg}$

Sol.

$$\text{Fats supplied} = 3.8 \times 10^7 \text{ J per kg}$$

$$\text{Efficiency rate} = 20\% \text{ or } \frac{20}{100}$$

$$\text{Efficient fats converted} = 3.8 \times 10^7 \times \frac{20}{100} = 7.6 \times 10^6 \text{ J}$$



$$\text{Work done by lifter 1 time} = mgh = 10 \times 9.8 \times 1 = 98 \text{ J}$$

$$\text{Work done to lift 1000 times} = \frac{98 \times 10^3}{1000} = 98 \text{ J}$$

$$\text{Fats used up} = \frac{\text{Total work}}{\text{Efficient fats converted}} = \frac{98 \times 10^3}{7.6 \times 10^6} = 12.89 \times 10^{-3} \text{ kg}$$

$$\boxed{\text{Ans: (B) } 12.89 \times 10^{-3} \text{ kg}}$$

Q) A particle experiences force given by  $\vec{F} = 3\hat{i} + 12\hat{j}$  undergoes displacement of  $\vec{d} = 4\hat{i}$ , particle had kinetic energy of 3J at beginning of displacement, what is its kinetic energy at end of displacement?

{JEE Mains 2019 (Online) 10<sup>th</sup> Jan Evening slot}

Sol.

$$\vec{F} = 3\hat{i} + 12\hat{j}$$

$$\vec{d} = 4\hat{i} + 0\hat{j}$$

$$W = \vec{F} \cdot \vec{d} = (3 \times 4)\hat{i} \cdot \hat{i} + (12 \times 0)\hat{j} \cdot \hat{j} = \underline{\underline{12J}}$$

$$W = 12J = \text{net work done}$$

$$\Delta KE = W = 12J \quad \{\text{Work-Energy Theorem}\}$$

$$\{KE_i = 3J (\text{given})\}$$

$$KE_f - KE_i = 12J$$

$$KE_f - 3 = 12J$$

$$\boxed{KE_f = 15J}$$

$$\boxed{\text{Answer : } \underline{15J}}$$

### Work, Power and Energy

(1) An object of mass  $1000\text{g}$  experiences a time dependent force  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})\text{N}$ . The power generated by the force at time  $t$  is:

Ans:-

$$\vec{F} = (2t\hat{i} + 3t^2\hat{j})\text{N}$$

$$m = 1000\text{g} = 1\text{kg}$$

$$\vec{F} = m\vec{a} ; \vec{a} = 2t\hat{i} + 3t^2\hat{j}$$

$$\therefore \frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$$

$$\Rightarrow \vec{v} = \int (2t\hat{i} + 3t^2\hat{j}) dt = t^2\hat{i} + t^3\hat{j} + \vec{C}$$

$$\therefore \vec{P} = \vec{F} \cdot \vec{v} = (2t\hat{i} + 3t^2\hat{j}) (t^2\hat{i} + t^3\hat{j})$$

$$\Rightarrow (2t \cdot t^2) + (3t^2 \cdot t^3) = P$$

$$\Rightarrow P = (3t^5 + 2t^3) \text{ W}$$

② A block of mass 25 kg is pulled along a horizontal surface by a force at an angle  $45^\circ$  with the horizontal. The friction coefficient between the block and the surface is 0.25. The block travels at a uniform velocity. The work done by the applied force during a displacement of 5m of the block is:

Ans:-

Block travels with uniform velocity.

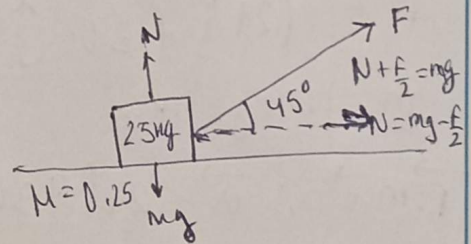
So,  $a = 0 \Rightarrow F \cos 45^\circ = \text{friction}$

$$\therefore \frac{F}{\sqrt{2}} = \mu [mg - \frac{F}{\sqrt{2}}]$$

$$\Rightarrow \frac{F}{\sqrt{2}} = 0.25 \left[ 25 \times 9.8 - \frac{F}{\sqrt{2}} \right]$$

$$\Rightarrow 1.25 \frac{F}{\sqrt{2}} = 61.25 \Rightarrow F = \frac{61.25 \times \sqrt{2}}{1.25} \Rightarrow F = 49\sqrt{2}$$

$$\therefore W_{\text{ext}} = FS \cos 45^\circ = 49\sqrt{2} \times 5 \times \frac{1}{\sqrt{2}} = 245 \text{ J}$$



③ A force  $\vec{F} = 2\hat{i} + b\hat{j} + \hat{k}$  is applied on a particle and it undergo a displacement  $\hat{i} - 2\hat{j} - \hat{k}$ . What will be value of  $b$ , if work done on particle is 0.

Ans:-

$$W = \vec{F} \cdot \vec{d}$$

$$\therefore \vec{F} = 2\hat{i} + b\hat{j} + \hat{k} ; \vec{d} = \hat{i} - 2\hat{j} - \hat{k}$$

$$\vec{F} \cdot \vec{d} = 2(1) + b(-2) + 1(-1) = 2 - 2b - 1 = 1 - 2b$$

$$\cancel{2} - \cancel{2}b - \cancel{1} = 1 - 2b \Rightarrow 1 - 2b = 0 \Rightarrow b = \frac{1}{2}$$

$\therefore$  value of  $b$  is  $\frac{1}{2}$ .

## WORK, ENERGY & POWER

1. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be

(a) 1:1 (b) 2:1 (c) 1:4 (d) 4:1

Answer:-  $K.E = \frac{1}{2}mv^2$

$$p = mv$$

$$v = \sqrt{\frac{2KE}{m}}$$

Sub in to momentum

$$p = m \sqrt{\frac{2KE}{m}} = \sqrt{2m \cdot KE}$$

For  $m_1 = 8 \text{ kg}$

$$p_1 \propto \sqrt{8} = 2\sqrt{2}$$

For  $m_2 = 2 \text{ kg}$

$$p_2 \propto \sqrt{2}$$

$$\frac{p_1}{p_2} = \frac{2\sqrt{2}}{\sqrt{2}} = 2$$

Ans:- (b) 2:1

2. A ball is projected with kinetic energy  $E$ , at an angle of  $60^\circ$  to horizontal. The kinetic energy of this ball at the highest point of its flight will become.

(a) zero (b)  $E/2$  (c)  $E/4$  (d)  $E$

Answer:-  $E = \frac{1}{2}mv^2$

initial speed  $v = \sqrt{\frac{2E}{m}}$

horizontal velocity =  $v_x = v \cos 60^\circ = \frac{v}{2}$

$$v_y = v \sin \theta = \frac{\sqrt{3}}{2}v$$

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \cdot m \cdot \frac{v^2}{4} = \frac{1}{8}mv^2 = \frac{E}{4}$$

option (c)  $E/4$

3. Bag of sand of mass  $9.8 \text{ kg}$  is suspended by a rope. A bullet of  $200 \text{ g}$  travelling with speed  $10 \text{ ms}^{-1}$  gets embedded in it, then loss of kinetic energy will be

(a)  $4.9 \text{ J}$  (b)  $9.8 \text{ J}$  (c)  $14.7 \text{ J}$  (d)  $19.6 \text{ J}$

Answer:  $KE = \frac{1}{2}mv^2$

$$\frac{1}{2}(0.2)(10)^2 = 10 \text{ J}$$

$$mu = (M+m)v$$

$$v = \frac{mu}{M+m} = \frac{0.2 \times 10}{9.8 + 0.2} = \frac{2}{10} = 0.2 \text{ m/s}$$

$$KE_{\text{final}} = \frac{1}{2}(M+m)v^2$$

$$\frac{1}{2}(10)(0.2)^2 = 0.5 \times 10 \times 0.04 = 0.2 \text{ J}$$

$$\Delta KE = KE_{\text{initial}} - KE_{\text{final}} = 10 - 0.2 = 9.8 \text{ J}$$

Option (b)

## Work Energy and Power

Q.1 An object of mass of 1000 g experiences a time dependent force  $\vec{F} = (2t\hat{i} + 3t^2\hat{j}) \text{ N}$ . The power generated by the force at time  $t$  is

Ans:  $m = 1000 \text{ g} = 1 \text{ kg}$

Using Newton's second law  $= \vec{F} = m\vec{a}$

Thus, acceleration,  $\vec{a} = 2t\hat{i} + 3t^2\hat{j}$

$$\frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$$

$$\vec{v} = (2t\hat{i} + 3t^2\hat{j}) dt = t^2\hat{i} + t^3\hat{j} + \vec{C}$$

(Assuming initial velocity is zero,  $\vec{C} = 0$ ,  $\vec{v} = t^2\hat{i} + t^3\hat{j}$ )

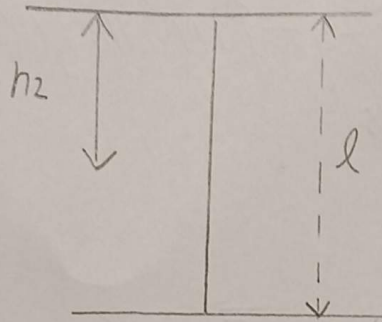
$$P = \vec{F} \cdot \vec{v} = (2t\hat{i} + 3t^2\hat{j}) \cdot (t^2\hat{i} + t^3\hat{j})$$

$$P = (2t \cdot t^2) + (3t^2 \cdot t^3) = 2t^3 + 3t^5$$

The power generated at time  $t$  is  $P = (2t^3 + 3t^5) \text{ W}$

Q.2 Length  $l$  of a chain of mass  $M$  hangs from a pulley. If it is wound such that half of the chain

remains overhung, the work done by the external agent is equal to



$$U_1 = m_1 g h_1$$

The final gravitational potential energy of the chain

$$U_2 = m_2 g h_2$$

$$U_2 = \left( \frac{m}{2} \right) g \left( \frac{l}{4} \right) = \frac{mgl}{8}$$

$\therefore$  The work done by the external agent

$$W = |U_1 - U_2|$$

$$\begin{aligned} W &= \frac{mgl}{2} \left( 1 - \frac{1}{8} \right) \\ &= \frac{3}{8} mgl \end{aligned}$$

Q.3 A ball falls under gravity from a height 10m with an initial velocity  $v_0$ . It hits the ground, loses 50%.

of its energy in collision and it rises to the same height. What is the value of  $V_0$ ?

$$\text{Ans: } V^2 - V_0^2 = 2g \times 10 = 2 \times 9.8 \times 10$$

$$V^2 = V_0^2 + 196$$

$$V'^2 - 0 = 2 \times 9.8 \times 10$$

$$V'^2 = 196$$

$$V' = 14 \text{ m/s}$$

Ratio of K.E before and after impact

$$= \frac{\frac{1}{2} m V^2}{\frac{1}{2} m V'^2} = \frac{V^2}{V'^2} = \frac{V_0^2 + 196}{196} = 2$$

$$V_0^2 = 2 \times 196 - 196 = 196$$

$$V_0 = 14 \text{ m/s}$$

## Work, Power & Energy.

Q1. rubber ball falls from height "h" and rebounds to h/2. The percentage loss of total energy of initial system as well as velocity of ball before it strikes down.

Sol "

velocity ~~by~~ before collision =  $\sqrt{2gh}$

• velocity after =  $\sqrt{2g(\frac{h}{2})}$

$$\therefore \Delta K.E = \frac{1}{2}m(2gh) - \frac{1}{2}mgh$$

$$= \frac{1}{2}mgh$$

$$\Rightarrow \frac{\Delta K.E}{K.E_i} \times 100 = \frac{\frac{1}{2}mgh}{\frac{1}{2}mg^2h} \times 100 = 50\%$$

$$50\% = \sqrt{2gh}.$$

Q2. A body of mass 50kg is lifted to a height of 20m from the ground in the two different ways as shown. The ratio of work done is equal to?

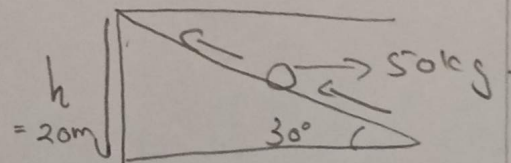
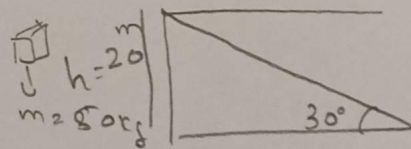
Sol:

$$W_1 = m \times g \times h$$

$$W_2 = m \times g \times h$$

$$\frac{W_1}{W_2} = \frac{1}{1} = 1/1$$

$$\boxed{1:1}$$



Q3. electron rotates in a circle around nucleus having charge  $Ze$ . find relation between total charge energy ( $E$ ) of electron to its potential energy.

Sol:

$$F = \frac{k(Ze)(e)}{r^2} = \frac{mv^2}{r}$$

$$k = \frac{1}{2} mv^2 = \frac{1}{2} \frac{(Ze)(e)}{r} k$$

$$U = \frac{k(Ze)(e)}{r}$$

$$E = \frac{k(Ze)(e)}{2r} - \frac{k(Ze)(e)}{r}$$

$$\Rightarrow \frac{-k(Ze)(e)}{2r}$$

$$E = U/2$$

$$\boxed{2E = U}$$

- Q. A force  $F = \alpha + \beta x^2$  acts on an object in the  $x$ -direction. The work done by the force is 5J when the object is displaced by 1m. If the constant  $\alpha = 1\text{N}$  then  $\beta$  will be. [JEE Main 2025]

Sol:  $F(x) = \alpha + \beta x^2$

$$W = \int_0^1 (\alpha + \beta x^2) dx$$

Subs  $\alpha = 1\text{N}$ :

$$W = \int_0^1 (1 + \beta x^2) dx = \int_0^1 1 dx + \int_0^1 \beta x^2 dx$$

$$\int_0^1 1 dx = [x]_0^1 = 1$$

$$\int_0^1 \beta x^2 dx = \beta \left[ \frac{x^3}{3} \right]_0^1 = \beta \left[ \frac{1^3}{3} - \frac{0^3}{3} \right] = \beta/3$$

Total Work done:

$$W = 1 + \beta/3$$

$$W = 5\text{J}$$

$$1 + \beta/3 = 5$$

$$\frac{\beta}{3} = 4$$

$$\underline{\underline{\beta = 12\text{N/m}^2}}$$

- Q. A body of mass 0.5kg travels on a straight line path with velocity  $v = (3x^2 + 4)\text{m/s}$ . The net work done by the force during its displacement from  $x = 0$  to  $x = 2\text{m}$  is:

Sol:  $v = 3x^2 + 4$

at  $x = 0$ ,  $v_1 = 4\text{m/s}$

$x = 2$ ,  $v_2 = 16\text{m/s}$

Work done =  $\Delta KE$

$$\frac{1}{2} \times m (v_2^2 - v_1^2)$$

$$= \frac{1}{2} (256 - 16)$$

$$= \underline{\underline{60\text{J}}}$$

Q. An object of mass 1 kg experience a time dependent force  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$  N. The power generated by the force at time  $t$ : [JEE Main 2025]

Sol:  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$  N, mass = 1 kg

$$\vec{F} = m\vec{a}$$

$$\therefore \text{acc} = \vec{a} = 2t\hat{i} + 3t^2\hat{j}$$

Velocity:

$$\frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$$

$$\vec{v} = \int (2t\hat{i} + 3t^2\hat{j}) dt = t^2\hat{i} + t^3\hat{j} + \vec{c}$$

$$P = \vec{F} \cdot \vec{v}$$

$$= (2t\hat{i} + 3t^2\hat{j}) \cdot (t^2\hat{i} + t^3\hat{j})$$

$$P = (2t \cdot t^2) + (3t^2 \cdot t^3)$$

$$= 2t^3 + 3t^5$$

Ans:  $2t^3 + 3t^5$  W

Q.) A Body of  $m = 1 \text{ kg}$  moves along a straight line with  $v = 2x^2$ . The work done by the body displacement from  $x = 0$  to  $5 \text{ m}$  is \_\_\_\_\_ J.

(A) 0

(C) 250

(B) 1250

(D) 1000

Ans:  $W = K_f - K_i$

At  $x = 0$ ,  $u_i = 0$

At  $x = 5$ ,  $u_f = 50$

$K_i = 0 \text{ J}$

$K_f = 1250 \text{ J}$

Ans (B) 1250 J

Q.) Which one of the following forces can't be expressed in terms of potential energy?

(1) Gravitational energy

(2) Friction ~~energy~~ force

(3) Coulomb force

(4) Restoring force

Ans:

(2) Friction force.

Q. An object of  $m = 1000\text{g}$  experiences a time dependent force  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})\text{N}$ . The power generated by force at time 't' is \_\_\_\_\_

Ans  $\vec{F} = (2t\hat{i} + 3t^2\hat{j})\text{N}$

$$m = 1\text{Kg}$$

$$\vec{F} = m \vec{a}$$

$$a \frac{d\vec{v}}{dt} = 2t\hat{i} + 3t^2\hat{j}$$

$$\vec{v} = t^2\hat{i} + t^3\hat{j}$$

$$P = \vec{F} \cdot \vec{v}$$

$$= (2t^3 + 3t^5)\text{W}$$

## WORK, POWER and ENERGY

Potential energy function for the force b/w two atoms in a diatomic molecule is approximately given by

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

where  $a$  and  $b$  are constant and  $x$  is the distance between the atoms. If the dissociation energy of the molec. is  $[U(x=\infty) - U \text{ at equil.}]$

$D$  is ?

$\Rightarrow U(x=\infty) = 0$ , at equil.

$$\frac{dU}{dx} = 0 \quad U(x) = \frac{a}{x^{12}} - \frac{b}{x^6} \longrightarrow (1)$$

$$12ax^{-13} - 6bx^{-7} = 0$$

$$12ax^{-13} = 6bx^{-7}$$

$$x^6 = 2a/b \longrightarrow (2)$$

From 1 and 2 :

$$U = \frac{a - b(2a/b)}{(2a/b)^2}$$

$$D = (0 - b^2)/4a$$

$$\boxed{D = b^2/4a.}$$

$$U = - b^2/4a$$

Q. A particle of mass  $m$  is moving in a circular path of constant radius  $r$  such that its centripetal acceleration  $a_c$  varying with time ' $t$ ' as  $a_c = k^2 r t^2$  where ' $k$ ' is a constant. The power delivered to the particle by the force acting on it is.

⇒ Centripetal acceleration ~~is~~  $a_c = k^2 r t^2$

$$v^2/r = k^2 r t^2$$

$$v^2 = k^2 r^2 t^2$$

$$mv^2/2 = m k^2 r^2 t^2 / 2$$

$$\text{kinetic energy} = m k^2 r^2 t^2 / 2$$

$$\frac{d}{dt} (KE) = m k^2 r^2 t$$

$$\boxed{\text{Power} = m k^2 r^2 t} \longrightarrow \underline{\text{Ans.}}$$

Q A body of mass  $m = 10^{-2} \text{ kg}$  is moving in a medium and experiences a frictional force  $F = -kV^2$ . Its initial speed is  $V_0 = 10 \text{ m/s}$ . If, after 10 s, its energy is the value of  $k$  will be

$$\Rightarrow \frac{1}{2} m v_1^2 = \frac{1}{8} m v_0^2$$

$$v_1 = v_0/2 = 5 \text{ m/s}$$

$$F = -k v^2 \longrightarrow (1)$$

$$F = ma = m (dv/dt) \longrightarrow (2)$$

From (1) and (2)

$$m (dv/dt) = -k v^2$$

$$\Rightarrow (dv/dt) = (-k/m) v^2$$

$$1/5 - 1/10 = -100k(10)$$

$$\boxed{k = 10^{-4} \text{ kg/m}}$$

Q1) A small block of mass 100g is pressed against a horizontal spring fixed at one end and compression is 5 cm. The spring constant is 100 Nm<sup>-1</sup>. When the block moves horizontally it leaves the spring. Where will it hit the ground 2 m below the string?

A) Let  $v$  be velocity when it leaves the spring.

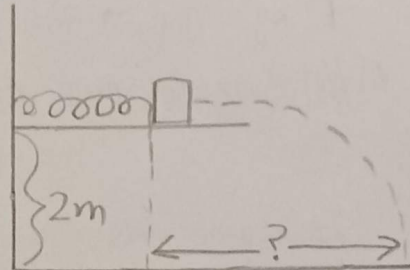
$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$v^2 = \frac{kx^2}{m} = \frac{100 \times 0.05^2}{0.1} = 0.05^2 \times 10^3$$

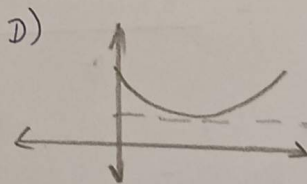
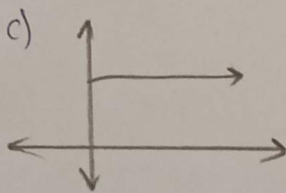
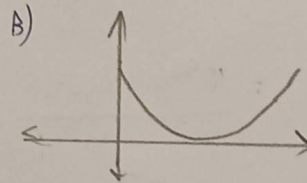
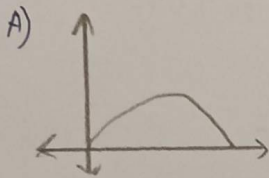
$$v = 10 \times 0.05\sqrt{10}$$

$$\text{Time to fall} = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}}$$

$$\text{Horizontal distance} = \text{time} \times v = \frac{2}{\sqrt{10}} \times 10 \times 0.05\sqrt{10} = 1 \text{ m} //$$



Q2) A projectile is fired with some velocity making certain angle with the horizontal. Which of the following graphs is the best representation for the kinetic energy of the projectile (KE) versus its horizontal displacement ( $x$ )?



A) Velocity of projectile at  $t$  instant

$$\Rightarrow v^2 = v_x^2 + v_y^2 = (v \cos \theta)^2 + (v \sin \theta - g \frac{x}{v \cos \theta})^2$$

$$K.E. = \frac{1}{2}mv^2 = mgx \tan \theta + \frac{gx^2}{v^2 \cos^2 \theta}$$

Above equation is parabolic.

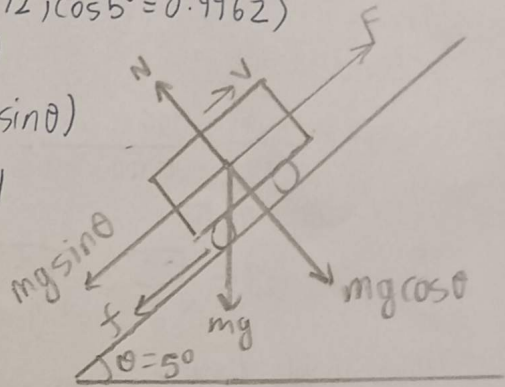
Answer cannot be B/A, cuz K.E. cannot be zero.

Answer: D //

Q3) A 100000 kg engine is moving up a slope of gradient  $5^\circ$  at a speed of 100 m/hr. The coefficient of friction between the engine and rails is 0.1. If the engine has efficiency of 4% for converting heat into work, find amount of coal, engine has to burn in 1 hour.

(1 kg coal  $\rightarrow$  50000 J) ( $\sin 5^\circ = 0.0872$ ,  $\cos 5^\circ = 0.9962$ )

A) Net force to move up,  $F = \mu n + mg \sin \theta$   
 $= mg(\mu \cos \theta + \sin \theta)$



If engine has to apply upward force equal to  $F$ , Power,  $P = Fv$   
 $v = 100 \text{ m/hr}$

Work done by engine,  $W = Pt = Fvt$

Efficiency of engine,  $\eta = \frac{\text{Output}}{\text{Input}} = \frac{Fvt}{\text{Energy Used}}$

Energy used  $= \frac{Fvt}{\eta}$

$= \frac{mg(\mu \cos \theta + \sin \theta) vt}{\eta}$  ( $\eta = \frac{4}{100} = 0.04$ )

$= \frac{10^5 \times 9.8 (0.1 \times 0.9962 + 0.0872) \times 100}{0.04}$

$= \frac{9.8 \times 0.1868 \times 10^7}{4}$

$= 4.577 \times 10^8 \text{ J}$

As 1 kg  $\Rightarrow$  50000 J

Amount of coal needed  $= \frac{4.577 \times 10^8}{5 \times 10^4} = 9154 \text{ kg//}$